TECHNICAL REPORT

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Title: Two Fundamental Equations for IR Ramp Fitting		Doc #: JWST-STScI-002161, SM-12 Date: July 7, 2010 Rev: -
Authors: M. Robberto	Phone: 410 - 338-4382	Release Date: 14 October 2010

1. Abstract

I provide two fundamental equations for ramp fitting of JWST data: 1) the equation to estimate the noise associated with a group of frames, to be used to fill in the noise map of individual groups in a ramp, and 2) the equation to estimate the noise associated with the count rate image obtained through ramp fitting. The first equation can be used e.g. for cosmic ray identification with thresholds varying with the signal, the second one represents the fundamental pixel noise that gets propagated through the pipeline up to the finally drizzled images.

2. Introduction

One of the earliest processing steps of the JWST data pipeline is to associate to each raw pixel sample a noise value. These noise values are used e.g. for cosmic ray identification or for the convergence of the linearization process. After line fitting, a second noise map has to be derived, relative to the count rate image. This eventually propagates through the entire pipeline up to the final noise maps of the drizzled images. It is therefore critical for the pipeline to properly estimate the noise associated with both the data cube and the final frames.

For the noise associated with the raw frames, it is clear that one cannot only consider the readout noise, as photon and dark current noises also contribute to the uncertainty. One can add the associated photon/dark current noise, but when different frames are combined together to estimate the signal rate one has to take into account that the photon noise terms associated with a sample are going to reappear in the following samples. In other words, the presence of signal adds correlation to the noise of a ramp. Furthermore, JWST uses group averaging, and therefore each group has its own internal correlation. Finally, one should also add the extra noise terms related to the bitshift error. In a previous report (Robberto 2009) I have laid down the theory to account for these terms. In this report I provide the equations to be used for the noise associated with the JWST grouped data and count rate images, adding the bitshift/quantization error that was not included in the earlier study.

3. Group average noise

Let's start with the most fundamental quantity one wants to obtain: the ramp slope *b*. It can easily be shown that the slope of a ramp made of *n* samples can be give as a weighted average of the individual samples, s_i.

$$b = \sum_{i=1}^{n} a_i s_i \tag{1}$$

For the weights, a_i, one may either assume the conventional expression for linear ramp fitting, i.e. Eq. (7) of Robberto (2009):

$$a_i = \frac{12}{dt \cdot n\left(n^2 - 1\right)}\left(i - \frac{n+1}{2}\right) \qquad (2),$$

or a more refined scheme like the one proposed by Regan (2007). The weights can be considered fully deterministic, i.e. they have no random error. Even if in the case of Regan 2007 the weights depend on the signal slopes, which are know with a certain error, we can assume the errors on those weights to be negligible.

Since the slope is calculated by combining different samples, its error $Var\left[b\right]$ must be estimated taking into account the covariance terms between the samples, i.e. (see Robberto 2009)

$$Var\left[b\right] = \sum_{i=1}^{n} a_i^2 Var\left[s_i\right] + 2\sum_{i=2}^{n}\sum_{j=1}^{i-1} a_i a_j Cov\left[s_i, s_j\right]. \tag{3}$$

In what concerns the individual sample (or, possibly, averaged group of samples) of index *i* the term of interest is $Var\left[s_i\right]$, because the covariance term enters only when different samples/groups are combined. In general, if *m* samples are averaged into a group and both photon/dark current and readout noise terms are taken into account, $Var\left[s_i\right]$ is given by Eq. (34) of Robberto (2009):

$$Var\left[s_i\right] = b\left(i-1\right)t_g + \frac{m+1}{2m} bt_f + 2\frac{1}{m^2}\sum_{k=2}^{m}\sum_{l=1}^{k-1} b\left(l-1\right)t_f + \frac{\sigma_{ron}^{\;\;2}}{m} \tag{4}$$

where *b* is the photon/dark current rate in units of e/s, t_f is the readout time for a single sample (typically 10.6s), t_g is the time required to read a group of frames (including both averaged and skipped frames), and σ_{ron} is the readout noise.

The term to be added to Equation (4) is the bitshift/quantization error, given by Robberto (2010):

$$Var[\text{e/adu}] = \left[\frac{g(\text{e/adu}) \times m}{\sqrt{12}}\right]^2. \tag{5}$$

where I indicate with g(e/adu) the conversion gain of the electronic chain ($g=2$ for NIRCam).

The final formula for the error of a sample/group can therefore be expressed as:

$$Var\left[s_i\right] = \frac{1}{12m}\left[g^2 m^3 + 4bt_f m^2 + 6b\left(2t_g(i-1) - t_f\right)m + 14bt_f + 12\sigma_{ron}^2\right]. \tag{6}$$

Table 1 shows the values obtained with Equation (6) by assuming 5 NIRCam readout patterns, a flux b=10e/s, a readout time t_f=10.6s and a readout noise σ_{ron}=20e. For comparison, I also show the results obtained using a "Basic" (and wrong) variance given by $b \cdot i \cdot t_g + \sigma_{ron}^2 / m$, i.e. an approximate value which does not take into account the correlation within groups and the bitshift error. It is clear that the "basic", intuitive, variance generally provides an overestimate, often very strong, of the single group variance. A cosmic ray removal algorithm based on sigma clipping that uses the "Basic" estimator would therefore miss a number of low energy cosmic ray events and produce a noisier image.

Table 1 Group Variance for a set of NIRCam readout patterns

	RAPID (m=1,s=0)		BRIGHT2 (m=2, s=1)		SHALLOW4 (m=4, s=1)		MEDIUM8 (m=8, s=2)		DEEP8 (m=8,s=12)	
i	Basic	Eq. (6)	Basic	Eq. (6)	Basic	Eq. (6)	Basic	Eq. (6)	Basic	Eq. (6)
1	506.000	506.333	518.000	280.833	630.000	224.583	1.110e3	316.458	2.170e3	316.458
2	612.000	612.333	836.000	598.833	1.160e3	754.583	2.170e3	1.376e3	4.290e3	2.436e3
3	718.000	718.333	1.154e3	916.833	1.690e3	1.285e3	3.230e3	2.436e3	6.410e3	4.556e3
4	824.000	824.333	1.472e3	1.235e3	2.220e3	1.815e3	4.290e3	3.496e3	8.530e3	6.676e3
5	930.000	930.333	1.790e3	1.553e3	2.750e3	2.345e3	5.350e3	4.556e3	1.065e4	8.796e3
6	1.036e3	1.036e3	2.108e3	1.871e3	3.280e3	2.875e3	6.410e3	5.616e3	1.277e4	1.092e4
7	1.142e3	1.142e3	2.426e3	2.189e3	3.810e3	3.405e3	7.470e3	6.676e3	1.489e4	1.304e4
8	1.248e3	1.248e3	2.744e3	2.507e3	4.340e3	3.935e3	8.530e3	7.736e3	1.701e4	1.516e4
9	1.354e3	1.354e3	3.062e3	2.825e3	4.870e3	4.465e3	9.590e3	8.796e3	1.913e4	1.728e4
10	1.460e3	1.460e3	3.380e3	3.143e3	5.400e3	4.995e3	1.065e4	9.856e3	2.125e4	1.940e4

4. Count rate noise

Equation (6) represents the correct general equation for the noise of an averaged group of samples. Values calculated according to this equation can be directly inserted in the noise maps of single or grouped raw JWST frames and be used, e.g., for CR identification.

Concerning ramp fitting, the slope can be estimated using Eq.(1), i.e. using deterministic weights a_i, and the error on the slope must be calculated using the correct expression to account for the correlation between frames. In the case of a uniformly sampled ramp, Equation (35) of Robberto (2009) provides the expression. The same study presents a Monte Carlo simulation showing that the equation exactly predicts the noise of the ramp, and also provides the expression for the more general case of weighted samples. However, Robberto (2009) does not include the bitshift/quantization error. Adding the bitshift/quantization error one has:

$$Var[b] = \frac{6}{5}\frac{(n^2+1)}{n(n^2-1)}\frac{b}{t_g}\left[1-\frac{5}{3}\frac{m^2-1}{m(n^2+1)}\frac{t_f}{t_g}\right] + 12\frac{1}{n(n^2-1)}\frac{1}{t_g{}^2}\left[\frac{\sigma_{ron}{}^2}{m}+\frac{g^2m^2}{12}\right] \qquad (7)$$

The way I have added the bitshift quantization error shows how this expression can be further generalized: uncorrelated noise terms that depend on the single read (e.g. noise associated with the reference pixel subtraction) can be added quadratically to the second term on the right-hand side as extra readout noise terms. Vice versa, correlated noise terms that depend on the integrated signal may be added to the first term.

As a final note, I remark that the algorithm works on linearized ramps. The linearization process adds some uncertainty that must be composed (added quadratically, assuming it is dominated by the uncertainty on the coefficients) to the error on the slope given by Equation (7).

5. Conclusions

In this report I have presented two fundamental equations for correct ramp fitting:

1) Equation (6), which gives the noise associated with an averaged group of frames. It can therefore be used to fill in the noise map of single or grouped raw frames in a ramp.
2) Equation (7), which gives the noise associated with the count rate image, obtained through ramp fitting.

6. References

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