TECHNICAL REPORT

Title: On the Linearity Correction of IR Ramps in the Case of Grouped Frames	Doc #: JWST-STScI-002588, SM-12 Date: 26 October 2011 Rev: -
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1.0 Abstract

I analyze the problem of correcting for the intrinsic non-linearity of IR detectors read non-destructively with co-added groups of frames. Theory shows that the linearity correction scheme I have recently introduced can successfully handle groups of averaged frames using a unique set of correction coefficients (per pixel). A test performed using data taken at the STScI ODL indicates that, for all NIRCam readout patterns, the "true" count rate of a ramp can be reconstructed within an error usually smaller than the theoretical noise floor associated with an ideally linear ramp with the same count rate.

2.0 Introduction

Hybrid IR focal plane arrays like those used by JWST are based on the "direct readout circuit" which allows for non-destructive signal sampling. This offers several advantages for e.g. cosmic-ray subtraction and readout noise reduction. On the other hand, it provides a response to the photon flux that is intrinsically non-linear and a correction for this effect has to be implemented by the data reduction pipeline. In the particular case of JWST, an extra complication is introduced by "group averaging", i.e. the co-addition/averaging of a number of adjacent frames to mitigate data volume issues. In two previous studies (Robberto 2010b, 2011) I have introduced a linearity correction algorithm that appears to largely improve over the scheme currently implemented by the HST IR instruments, NICMOS and WFC3. In this report I first consider theoretically the issue of the non-linearity correction with grouped frames and then validate the performance of the new algorithm with grouped frames using data taken at the STScI ODL.

3.0 Correcting for non-linearity with coadded frames

Non-linearity correction is needed to derive the ramp parameters, i.e. the intercept *a* and the slope (count rate) *b* from the measured counts C_i sampled at time t_i. The current pipeline for the two HST IR instruments, NICMOS and WFC3, performs this operation by multiplying the measured counts by a correction function F that depends on the measured counts themselves, i.e.

$$a + bt_i = c_i \cdot F(c_i)$$

The form of the function F is assumed to be polynomial up to some order in the measured counts, i.e.

$$a + bt_i = c_i\left(1 + H_0 c_i + H_1 c_i^2 + H_2 c_i^3\right), \tag{1}$$

where the coefficients H_0, H_1 and H_2 are derived with some calibration process.

In the case of JWST, most often one will deal with group-averaged counts. This means that c_i values will actually be given by averages of the real counts

$$g_i = \frac{1}{n}\sum_{j=1}^{n} c_i$$

sampled at mean time

$$\tau_i = \frac{1}{n}\sum_{i=1}^{n} t_i$$

It is clear that in this case the non-linearity of Eq. (1) prevents a direct derivation of the a and b coefficients: performing the non-linearity correction before (as would be appropriate) or after co-adding the frames gives different results. In the first case, "first correct – then coadd", Eq. (1) gives:

$$a + b\tau_i = \sum_{j=1}^{n} c_j \left(1 + H_0 \sum_{j=1}^{n} c_j + H_1 \sum_{j=1}^{n} c_j^2 + H_2 \sum_{j=1}^{n} c_j^3\right)$$

while in the second case, "first coadd - then correct", we deal with a different expression containing the squares and the cubes of the averaged or co-added frames:

$$\begin{aligned} a + b\tau_i &= g_i\left(1 + K_0 g_i + K_1 g_i^2 + K_2 g_i^3\right) \\ &= \sum_{j=1}^{n} c_j \left(1 + K_0\left(\sum_{j=1}^{n} c_{i,j}\right) + K_1\left(\sum_{j=1}^{n} c_{i,j}\right)^2 + K_2\left(\sum_{j=1}^{n} c_{i,j}\right)^3\right). \end{aligned}$$

This expression cannot be put in a form similar to Eq. (1); in principle, calibration is possible only if the K_0, K_1 and K_2 coefficients have been derived for the flux rate b one wants to measure.

In two previous reports, I have introduced a different way of correcting for non-linearity, "turning upside down" Eq. (1). The idea is to make F a function of incoming flux, rather than of the measured counts:

$$c_i = a + bt_i \cdot F\left(bt_i\right)$$
$$= a + bt_i\left[1 + H_0 \cdot bt_i + H_1 \cdot \left(bt_i\right)^2 + H_2 \cdot \left(bt_i\right)^3\right] \quad (2)$$

For simplicity I keep here for the coefficients the same *H* nomenclature used in Eq. (1), but it is evident that they are not those used in Eq. (1). I will not review here the advantages of this approach as they have been already illustrated in the two earlier reports. I will instead underscore that this formalism nicely allows correcting for the non-linearity of grouped frames. Expanding Eq. (2)

$$c_i = a + bt_i \cdot F\left(bt_i\right)$$
$$= a + bt_i + H_0\left(bt_i\right)^2 + H_0\left(bt_i\right)^3 + H_1\left(bt_i\right)^4$$

it is clear that coadding *n* frames one has

$$g_i = na + b\sum_{j=1}^{n} t_j + H_0 b^2 \sum_{j=1}^{n} t_j^2 + H_1 b^3 \sum_{j=1}^{n} t_j^3 + H_2 b^4 \sum_{j=1}^{n} t_j^4$$

which can be inverted using the same *H* parameters used for reconstructing the data taken without grouping. The key point is that here we deal with power-sums of the known readout times t_i, rather than with the unknown counts c_i.

4.0 Validation

To test the viability of this scheme, i.e. that the same *H* coefficients used for the non-linearity correction in the case $n=1$ also work when $n>1$, I have used the data provided by M. Regan for the "non-linearity shootout" (Regan 2011, private communication). Using my non-linearity correction algorithm, I have derived the *H* parameters from the data-cube

/grp/jwst/wit/odl/non_lin/high1_491_diff_cube.fits

which has been reference-pixel corrected and read_0 subtracted. The procedure derives the correction coefficients for every pixel of the array. Following Regan's recommendation, I concentrate the analysis on a few test pixels. In particular, I present here the results for the first in the list, $x=1941$, $y=892$ (in IRAF convention, i.e. counting from pixel $x=1$,$y=1$), other pixels providing similar results. Using the derived *H* parameters for this pixel, I have corrected the non-linearity of a second, independent datacube:

/grp/jwst/wit/odl/non_lin/high2_491_diff.fits

containing 59 frames sampled non-destructively up to the saturation level. In both the derivation of the *H* coefficients and in the reconstruction, I use data points departing up to ~20% from the reconstructed linear relation. The 59 frames are averaged 2 by 2, 4 by 4 and 8 by 8 and the resulting ramp corrected for linearity.

Dealing with real data, one should not expect an absolutely perfect reconstruction, i.e. to find exactly the same values for the reconstructed slopes, because of the presence of noise on data that are averaged and censored in different ways depending on the ramp. In particular, since at the end of the integration the last group may be incomplete and the ramp truncated, the reconstructions always use different "ingredients". The results are illustrated in Figures 1 to 4.

The case *n=1* (Figure 1) is the reference. Here it is obviously appropriate to correct using the default H parameters. The derived slope is *b=69.597* c/s and the reconstruction error, defined as the difference between the measured, non-linear data points and the values calculated by applying "back" the polynomial perturbation (Eq. 2) to the estimated count rate, is completely negligible. This is generally an indication of good convergence of the reconstruction algorithm.

In the case *n=2* (Figure 2), the derived slope is *b=69.609* c/s, a departure of about 0.004% from the case *n=1* after correcting the ramp up to ~20% departure from linearity.

In the case *n=4* (Figure 3), the derived slope is *b=69.283* c/s and the discrepancy from the case *n=1* increases to 0.11%.

In the case *n=8* (Figure 4), the derived slope is *b=68.956* c/s with a discrepancy of 0.23%.

Similar results can be obtained for the great majority of pixels; in general, the ramps are reconstructed up to 20% with less than 1% error, which can be regarded as an excellent result. The last step of this investigation consists in validating these finding for the specific NIRCam readout patterns.

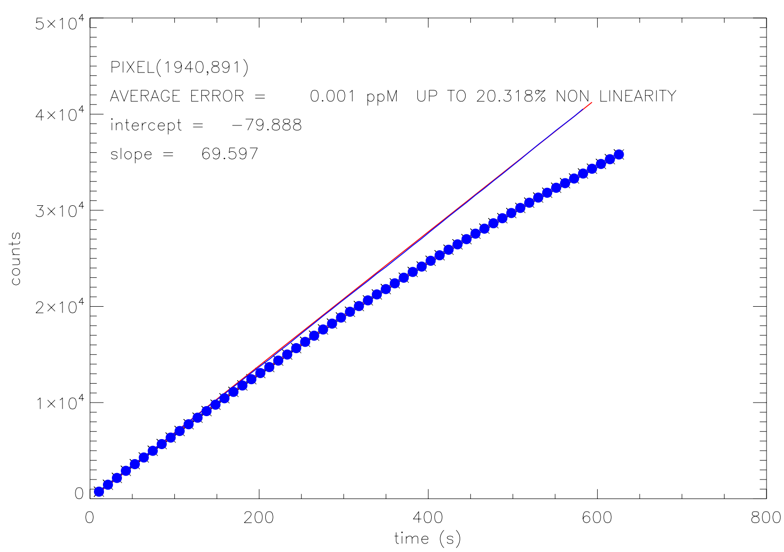

Figure 1: Ramp for pixel (1941, 892), corresponding to pixel (1940, 891) in IDL convention, before and after linearization. In this case it is N_{group}=1 and 56 data points are used (the last 3 are rejected because they depart more than 20% from the linear regime).

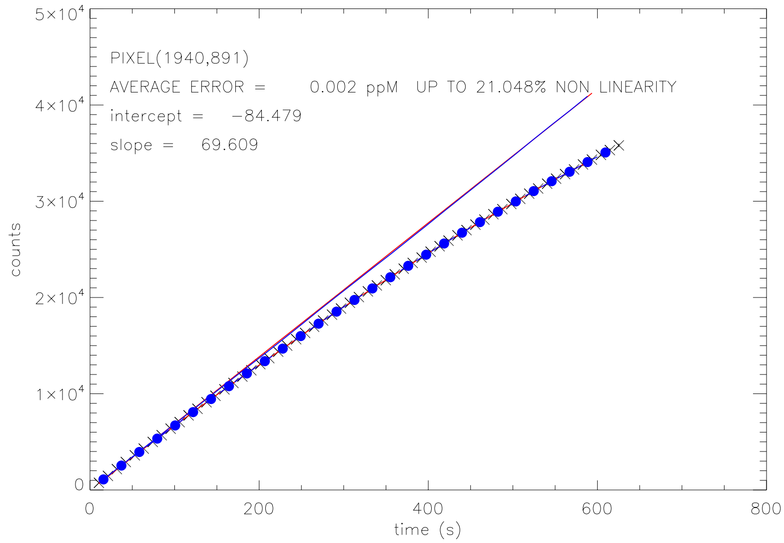

Figure 2: Same as Figure 1 for Ngroup=2. The crosses indicate the original data points, used in Figure 1. The blue circles are the 29 average values used for the linearity reconstruction. The red line represents the reconstruction obtained with Ngroup=1; the blue lines, almost exactly superposed to the red ones, represent the reconstruction with Ngroup=2.

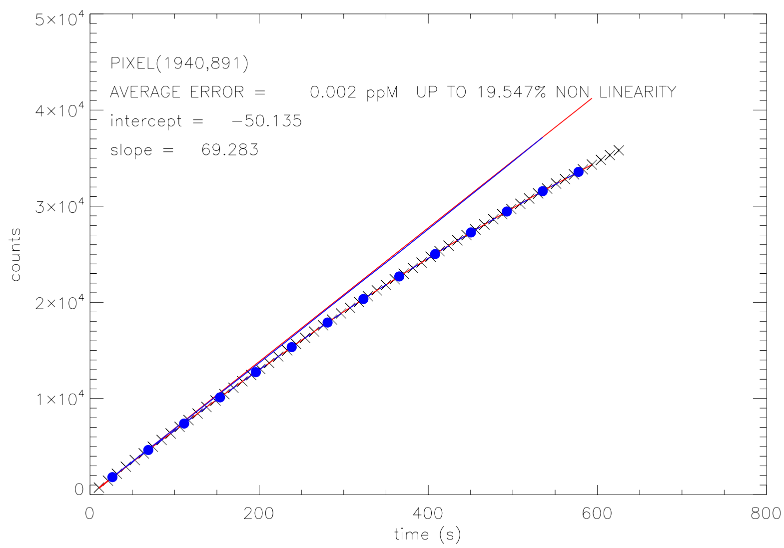

Figure 3: Same as Figure 1 for Ngroup =4. The crosses indicate the original data points, used in Figure 1. The blue circles are the 14 average values used for the linearity reconstruction. The red line represents the reconstruction obtained with Ngroup =1; the blue lines, almost exactly superposed to the red ones, represent the reconstruction with Ngroup =4.

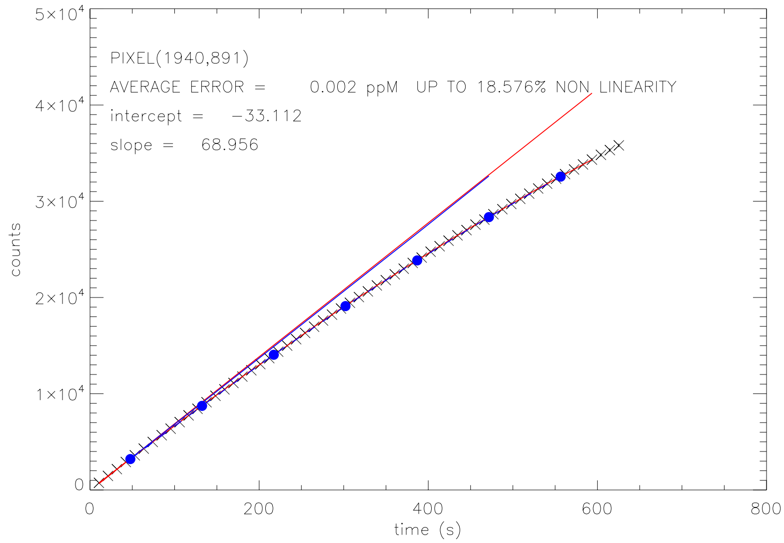

Figure 4: Same as Figure 1 for Ngroup =8. The crosses indicate the original data points, used in Figure 1. The blue circles are the 7 average values used for the linearity reconstruction. The red line represents the reconstruction obtained with Ngroup =1; the blue lines, almost exactly superposed to the red ones, represent the reconstruction with Ngroup =8.

There are 9 NIRCam readout patterns, 7 of them characterized by the introduction of missing (skipped) frames between groups. Table 1 lists the pattern parameters and the relative reconstruction results. The longest ramps, Deep2 and Deep8, have been adjusted to match the relatively low number (59) of test frames available: N_{skip} has been reduced by 2 frames, from 18 to 16 and from 12 to 10. This allowed setting the number of groups, N_{group},to 3, the minimum required by the current version of the algorithm to converge.

I have included in Table 1 also the error associated with the estimated slope, calculated using the general expression for the error of the slope derived in Robberto (2010a). This is the theoretical error, in count/seconds, associated with a ramp of slope *b* (counts/seconds) sampled with given values for N_{group}, N_{skip} and N_{frames}. I have assumed a readout noise for a single read equal to 13e, corresponding to 18e noise in Correlated Double Sampling. I have neglected the digitization noise, since the averages are performed at 32 bit, and no attempt to account for 1/f noise has been made. Basically, this is the noise expected if that particular ramp had been created intrinsically linear; it therefore represents a “floor” level beyond which one cannot go, unless one reduces the electronic readout noise of the single read.

Table 1 shows that all *b* values for the reconstructed linear slopes fall within 1sigma of the “true” value b=69.6597 measured earlier using the full set of individual frames. The linearity correction scheme can therefore successfully reconstruct all NIRCam ramps adding a relatively modest, often negligible, amount of error. Again, other pixels provide similar results.

Table 1: Slope of the ramp for pixel 1941,892 for each NIRCam readout pattern.

Readout Pattern	Ngroup	Nskip	Nframes	slope	error
Rapid	1	0	10	68.01	1.63
Bright1	1	1	10	68.49	1.56
Bright2	2	0	10	68.35	1.18
Shallow2	2	3	10	69.21	1.09
Shallow4	4	1	10	69.33	0.81
Medium2	2	8	10	69.44	1.05
Medium8	8	2	10	69.30	0.57
Pseudo-Deep2	2	16 [18]	3	68.51	6.51
Pseudo-Deep8	8	10 [12]	3	68.23	3.27

5.0 Conclusions

I have analyzed the problem of correcting for the intrinsic non-linearity of IR detectors when frames are co-added or averaged in groups. The linearity correction scheme I have recently introduced successfully handles groups of averaged frames using a unique set of correction coefficients (per pixel), i.e. the optimal coefficients derived for the ideal case of long ramps sampled with single frames. A test performed using data taken at the STScI ODL indicates that, for all NIRCam readout patterns, the "true" count rate of a ramp can be reconstructed with discrepancies that are usually smaller than the theoretical noise floor associated with an ideally linear ramp obtained in the same conditions (same count rate and same readout pattern).

6.0 References

Robberto M., Two Fundamental Equations for IR ramp fitting, JWST-STScI-002161 (2010a)

Robberto M., An improved algorithm for the correction of IR detector non-linearity, JWST-STScI-002163 (2010b)

Robberto M., Implementation of a new algorithm for the correction of non-linearity in JWST detectors, JWST-STScI-002344 (2011)